

Day and Date:day, .../.../2022

Time: ----- to -----

Max. Marks- 100

Instructions:

1. Question No. 1 is compulsory.
2. Figure to the right indicate **full marks**.
3. Use of **non-programmable** calculator is allowed.

BT	CO's	Q.No.		Marks
		Q.1	Attempt the following	40
L3	101.1	a	i) Test for consistency and if consistent solve $x_1 + 2x_2 + 2x_3 = 1, 2x_1 + 2x_2 + 3x_3 = 3, x_1 - x_2 + 3x_3 = 5.$	5
			ii) Solve by Gauss elimination method $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$	5
L3	101.1	b	i) Examine for linear dependence or independence of vectors $X_1 = [1, 1, 1, 3], X_2 = [1, 2, 3, 4], X_3 = [2, 3, 4, 7]$	5
			ii) Evaluate Eigen values and Eigen vector corresponding to largest Eigen vale for the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	5
L3	101.1	c	i) Solve $x^6 - i = 0$	5
			ii) Show that $\cos 5\theta = 5 \cos \theta - 20 \cos^3 \theta + 16 \cos^5 \theta$	5
L3	101.2	d	i) Reduce the following matrix to normal form and calculate the rank for $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$	5
			ii) Compute the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Make use of it to calculate A^{-1} .	5

		Q.2	Attempt any four from the following	20
L3	101.4	a	Prove that $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \dots$	5
L3	101.4	b	Using Taylor's theorem express $7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4$ in power of x	5
L3	101.4	c	Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$	5
L3	101.4	d	Evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - (1 + x)^2}{x \log(1 + x)}$	5
L3	101.4	e	Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	5
		Q.3	Attempt any four from the following	20
L3	101.4	a	If $z = f(x + ay) + \phi(x - ay)$, prove that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.	5
L3	101.4	b	If $u = \frac{x^3 y + y^3 x}{y - x}$. Prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 6 \left[\frac{x^3 y + y^3 x}{y - x} \right].$	5
L3	101.4	c	If $x = r \cos \theta, y = r \sin \theta$. Prove that $J J' = 1$	5
L3	101.4	d	Discuss the maximum and minimum of $x^3 + y^3 - 3axy, a > 0$.	5
L3	101.4	e	If $z = x^x y^y$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.	5
		Q.4	Attempt any four from the following	20
L3	101.3	a	Apply Bisection method to solve $x^3 - x - 1 = 0$ up to fifth iterations, whose root lies between 1 and 2.	5
L3	101.3	b	Use False position method to calculate the root of equation up to fourth iterations for $\cos x - x e^x = 0$ (Given that root lies between 0 and 1)	5

L3	101.3	c	Calculate the root of equation $x^3 + x - 1 = 0$ using Newton-Raphson method with initial guess value $x_0 = 0$	5
L3	101.3	d	Make use of Secant method to find the root of equation $x^3 - 2x - 5 = 0$ between 2 and 3 up to fourth iteration.	5
L3	101.3	e	Calculate the positive root of equation $3x - \cos x - 1 = 0$ using Newton-Raphson method using $x_0 = 1$.	5